Optimal Signal Timing for Multi-Phase Intersections

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**Aim of the research:** Develop an optimal timing policy for multi-phase intersections, allowing for prioritized traffic.

**Motivation:**

- Most of the optimal methods only apply to simple intersections (two phases).
- Policies that assign higher priority to low-speed vehicles (buses, trucks,...) can improve the overall traffic state.
Store-and-forward model

Oversaturated condition

\[ q_{j,T+1} = q_{j,T} + C a_j, T - \left( \sum_{i=k_j}^{l_j} g_{i,T} \right) d_j, T \]

Sum is done over all phases that serve this lane group.

- \( q_{j,T} \) [veh] queue length of lane group \( j \) at beginning of cycle \( T \).
- \( a_j, T \) \((d_j, T)\) [veh/s] arrival (departure) rate for lane group \( j \) at cycle \( T \).
- \( g_{i,T} \) [s] green time for phase \( i \) at cycle \( T \).
Store-and-forward model
Undersaturated condition

\[
q_{j,T+1} = q_{j,T} + C a_{j,T} - \left( \sum_{i=k_j}^{l_j} g_{i,T} - z_{j,T} \right) d_{j,T} - z_{j,T} a_{j,T}
\]

Sum is done over all phases that serve this lane group.

- \( q_{j,T} \) [veh] queue length of lane group \( j \) at beginning of cycle \( T \).
- \( a_{j,T} \) \((d_{j,T})\) [veh/s] arrival (departure) rate for lane group \( j \) at cycle \( T \).
- \( g_{i,T} \) [s] green time for phase \( i \) at cycle \( T \).
- \( z_{j,T} \) [s] zero-queue-length period.
The consideration of both conditions, **oversaturated** and **undersaturated**, results in the **nonlinear** model:

\[
q_{j,T+1} = q_{j,T} + C_{aj,T} - \left( \sum_{i=k_j}^{l_j} g_{i,T} - z_{j,T} \right) d_{j,T} - z_{j,T} a_{j,T}
\]

\[
z_{j,T} = \max \left( 0, \sum_{i=k_j}^{l_j} g_{i,T} - \frac{q_{j,T} + \left( \sum_{i=1}^{k_j-1} g_{i,T} \right) a_{j,T}}{d_{j,T} - a_{j,T}} \right)
\]
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In the view of the previous facts, the maximization of the weighted throughput within one signal cycle is a nonlinear problem.

$$\max_{g_i,T} \sum_{j=1}^{J} w_j, T (q_{j,T+1} - q_{j,T})$$

Sum is done over all lane groups.
Throughput maximization via Linear Programming

It is possible to relax the nonlinear problem by adding $J$ new decision variables $\bar{z}_{j,T}$:

$$\bar{z}_{j,T} \geq \sum_{i=k_j}^{l_j} g_{i,T} - \frac{q_{j,T} + \left(\sum_{i=1}^{k_j-1} g_{i,T}\right) a_{j,T}}{d_{j,T} - a_{j,T}},$$

$$\bar{z}_{j,T} \geq 0,$$

It can be proved that the optimization problem may be posed as a linear programming (LP) with $I + J$ decision variables.

$I$ ($J$) is the number of phases (lane groups) in the intersection.
Time-invariant scenario

Assuming $d_{j,T} = d_j$, $a_{j,T} = a_j$, $w_{j,T} = w_j$ for all $T$, the queue lengths dynamic is given by a linear time-invariant (LTI) system

$$q_{T+1} = q_T + Bu_T,$$

where $u_T = [g_{1,T} \cdots g_{I,T} \ z_{1,T} \cdots z_{J,T}]^\top$ and $q_T = [q_{1,T} \cdots q_{I,T}]^\top$.

Using multi-parametric linear programming (mp-LP), it is possible to obtain

$$u_T(q_T) = A_m q_T + b_m \quad \text{if} \quad q_T \in \mathcal{R}_m$$

An analytical expression for the green times as function of the queue lengths.
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Intersection of Two One-Way Roads

\[ d_1 = 0.55 \text{ veh/s}, \quad d_2 = 0.60 \text{ veh/s}, \]
\[ a_1 = 0.10 \text{ veh/s}, \quad a_2 = 0.15 \text{ veh/s}, \]
\[ g_{1,\text{min}} = 5 \text{ s}, \quad g_{2,\text{min}} = 5 \text{ s}, \]
\[ w_1 = 1, \quad w_2 = 1, \quad C = 30 \text{ s}. \]

\[
\begin{bmatrix}
    g_1, T \\
    g_2, T
\end{bmatrix} =
\begin{bmatrix}
    0.00 & -1.67 \\
    0.00 & 1.67
\end{bmatrix}
q_T +
\begin{bmatrix}
    22.50 \\
    7.50
\end{bmatrix}
\]

Figure shows queue lengths trajectories for some initial conditions, all trajectories evolve to the same equilibrium point \( q_{eq} = \begin{bmatrix} 0.75 & 0.00 \end{bmatrix}^T \).
Multi-Phase Intersection

Major intersection of Abba Khoushy Avenue and Oscar Schindler Street in Haifa, Israel.

This intersection is modeled in Aimsun microsimulation.
Multi-Phase Intersection
Comparison between prioritized policy for buses and current control plan used by the Haifa municipality

<table>
<thead>
<tr>
<th></th>
<th>Avg. delay time, s/km</th>
<th>Avg. number of stops, #/veh/km</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cars</td>
<td>327.08</td>
<td>3.20</td>
</tr>
<tr>
<td>Buses</td>
<td>414.84</td>
<td>3.81</td>
</tr>
</tbody>
</table>

Evaluation of the current control plan.

Evaluation of the traffic light policy with different priority levels for buses.

Simulations in other scenarios shown similar results.
Optimal Signal Timing for Multi-Phase Intersections

Conclusions

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Conclusions

- The proposed policy is very simple to implement, it only requires one LP at each signal cycle.
- The policy can be expressed as an analytical expression when assuming a time-invariant scenario.
- Overall traffic conditions may be improved when applying the proposed policy and assigning higher priority to buses.
Thank you for your attention