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USING AN OFFICIAL UNDULATION MODEL FOR ORTHOMETRIC HEIGHT ACQUISITION BY GNSS

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ABSTRACT

Since the beginning of 2006 the vertical control system in Israel has been ellipsoidal. The Survey of Israel constructs and maintains the ellipsoidal vertical control system based on permanent Global Navigation Satellite System (GNSS) stations. Nowadays, the vertical control system's objective of providing a framework upon which topographers and engineers can base and adjust their heights is achieved more easily by GNSS measurements. Despite this, the vertical orthometric control system is still in use since many height data users prefer the orthometric system. However, the characteristics of the orthometric control system are localized.

The Geoid Undulation Model is one of the foundations of the geodetic infrastructure. It is used to connect ellipsoidal and orthometric heights. Intensive utilization of GNSS for geodetic, surveying and engineering applications necessitates the rapid development of an undulation model. Today, the efforts to develop a geoid undulation model with an accuracy level of one centimetre over an entire country demand multiple resources. This paper suggests a method for developing an orthometric control system with a reasonable accuracy level on a nationwide basis by means of GNSS measurements, ellipsoidal vertical control and an official geoid undulation model. The best available geoid undulation model can be used as the official geoid undulation model, regardless of its accuracy level.

Research was conducted in Israel in order to test the feasibility of the idea of using an official geoid undulation model. Two kinds of official models were tested, the world wide geopotential model GPM98B and a local countrywide model computed by the Survey of Israel. We compared the orthometric height differences obtained by GPS measurements using the undulation model with the known orthometric differences. This paper presents the results of these experiments. It verifies the capability of the suggested technique to define seamless orthometric vertical control, adequate for most geodetic and surveying purposes, via a low-cost and fast procedure.

KEYWORDS: Undulation model. Orthometric height. Ellipsoidal height. GNSS.

INTRODUCTION

Levelling is the traditional geodetic method for observing elevation differences between points on the Earth's surface. It is indeed an accurate measuring method, but it is notoriously time-consuming, very expensive in terms of manpower, and extremely cumbersome in its requirement for literally covering the space between every pair of end-points, step-by-step. Due to the nature and practical limitations of levelling, most vertical control points are located in valleys, along roads and railways. In contrast, horizontal control networks have been historically established on hilltops or high points. As a result, most countries have completely separate networks for horizontal and vertical control. Nowadays, Global Navigation Satellite Systems (GNSS) enable the creation of accurate spatial (3D) control networks.

The geoid undulation (N) is the separation between the geoid and the ellipsoid. It is required for many geodetic and surveying applications. Its most notable use is the transformation between ellipsoidal heights and orthometric heights.

The geoid can be broadly defined as the equipotential surface of the Earth's gravity field that corresponds most closely with mean sea level [**3**]. The geoid forms the reference surface for orthometric heights.

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GNSS provides the ability to specify the exact spatial location of a point. The location is referenced directly to the WGS84 (World Geodetic System 1984) datum for GPS, and the SGS85 (Soviet Geodetic System 1985) for GLONASS. The height of a point is referred to the ellipsoid surface. Therefore WGS84 (or SGS85) forms the reference surface for ellipsoidal heights. The connection between ellipsoidal height (h) and orthometric height (H) is achieved (approximately) by [**3**],

$$h = H + N \tag{1}$$

Hence, ellipsoidal heights can be used to determine orthometric heights if the geoid undulation (N) is known. Equation (1) is not exact because it ignores the deflection of the vertical.

In the relative case, an orthometric height difference (ΔH) between two points, *i* and *j,* can be described according to Eq. (1) as:

$$\Delta H_{ij} = (h_i - N_i) - (h_j - N_j) = \Delta h_{ij} - \Delta N_{ij} \tag{2}$$

This procedure provides a viable alternative over conventional levelling methods. GNSS levelling, as a mean for establishing orthometric height differences with respect to a local vertical datum, is dependent on the achievable accuracy of the ellipsoidal heights and the geoid undulations data. The geoid undulation model is one of the corner stones in the geodetic infrastructure. Enormous efforts are undertaken around the world in order to achieve accurate geoid models. The general attitude is that we can not use GNSS measurements in order to achieve accurate orthometric heights, unless we use a high-accuracy geoid model. Therefore, the general aim is for the most accurate geoid model possible. This paper claims that for most surveying tasks requiring orthometric control networks, a permanent GNSS network and an existing undulation model that serves as the official geoid undulation model can suffice for converting ellipsoidal heights to orthometric heights and can provide a high level of accuracy.

VERTICAL CONTROL NETWORKS

The objective of vertical control networks is to provide a framework on which topographers and engineers can base and adjust their heights [**2**]. For the common surveying and engineering needs, the actual necessary accuracy of height differences is about 10 mm to a distance of 100 m (a relative accuracy of 100 ppm). However, due to the nature of levelling, one might make a gross mistake in his work and still not find any abnormal misclosure once one has conducted one-way levelling. Therefore, the actual accuracy for vertical control should be better than the relative accuracy of 100 ppm. In General, a vertical control network for surveying and engineering work would be considered reasonable if it consisted of benchmarks with a local accuracy (including internal stability) of approximately 1-2 cm, and an accuracy of approximately 25 mm in the height difference between two benchmarks one kilometre apart, which is a relative accuracy of about 25 ppm (see also [**4**]).

Orthometric Control

Orthometric heights are based on the geoid as a reference surface. The classic vertical orthometric control is composed of several hierarchical networks that follow the principle "from the whole to the part". The primary framework should be a net of high precision levelling lines. The accuracy of precise levelling should be at the millimetre-level per kilometre [**2**]. The other subnets in the network are densifications of the primary framework, according to need, and with a decreasing accuracy.

A lot of work was carried in Israel in order to establish an accurate and homogenous vertical control network based on precise levelling. The Survey of Israel (SOI) maintained a first/second order network with loops misclosure of less than $3\text{mm}/\sqrt{\text{km}}$ all over the country, and a third order network with line misclosure of less than $15\text{mm}/\sqrt{\text{km}}$, only in densely populated parts of the country.

It is almost impossible to get heights for lower-order networks with an absolute accuracy that is better than 5-10 cm relative to the higher-order network. The final achievable accuracy depends on the distance and the height differences of the low order benchmarks from the higher order benchmarks. For this reason many local vertical orthometric control networks are established. Height discrepancies between neighbouring benchmarks of two different local systems often reach up to 10 cm.

Ellipsoidal Control

Ellipsoidal heights are based on the defined ellipsoid and are purely geometric. Ellipsoidal control may be the forthcoming replacement for the orthometric control. The potential of using ellipsoidal heights in non-engineering and engineering applications was discussed and explained by Kumar [**5**]. In the United States, for example, the National Geodetic Survey will not create or maintain benchmarks by levelling. Vertical control is assigned by estimating orthometric heights from ellipsoidal heights, as computed from GPS measurements [**7**]. The Survey of Israel has already decided to move towards a 3D geodetic control network, based on the Israeli permanent GNSS network [**11**]. In this case the vertical ellipsoidal control is one part of the 3D geodetic control that is based on the permanent GNSS network. Ellipsoidal control should have a hierarchical form [**13**]. The permanent GNSS network is then used as the first order of the 3D control network. Usually the locations of permanent stations are quite far from each other (on a scale of tens of kilometres).

The GPS baseline accuracy is dependent on the vector length. Therefore, it is recommended to increase the density of the first-order control network by using more points, according to need. Naturally, the densification should be accomplished by GNSS measurements. The relative vertical precision (one sigma) of a permanent GNSS site is about 3 mm [**8**], [**1**]. The SOI decided that the accuracy of the second-order network will be 1 cm (2-sigma), and 2 cm for the third-order, relative to the nominal heights of the permanent GNSS stations.

GEOID UNDULATION MODELS

One of the primary tasks of geodesy is to develop a geoid model [**16**]. One of its uses is to connect the ellipsoidal height to the orthometric height. Given N, we can compute H or h. It is well understood that the higher the accuracy of the geoid undulation model is, the more accurate the conversion of ellipsoidal heights into orthometric heights. In general, the global or large-scale features of the geoid are expressed by a spherical harmonic expansion of the gravitational potential. Global models have a level of accuracy of a decimetre worldwide [**6**], [**15**]. However, it is important to note that the accuracy of any geoid model is a function of its location on earth.

A high-resolution and accurate geoid model may be derived for a local area or even nationwide. Local models can have an accuracy of 1 centimetre [**9**]. An accurate geoid of the State of Israel does not exist, except for a small area (about 600 square kilometres) on and around the Carmel Mountains in the northern part of Israel [**10**].

In recent years, the Survey of Israel made relative GPS measurements between the points of the primary vertical orthometric control network in Israel. These provided WGS84 ellipsoidal heights for about 700 points covering most of Israel. Subtracting orthometric heights from WGS84 ellipsoidal heights at these points yielded discrete estimates of the separation between the local Israel vertical datum and the WGS84 ellipsoid, which can be interpolated to form a geometrically-derived geoid model. Kriging interpolation was used to compute the local Israeli geoid undulation model, where over most of the country the accuracy of the model is better than 10 cm [**14**].

GPM98B is a global geopotential model, which approximates the Earth's gravity field and computes geoid heights in terms of spherical harmonics to degree 1800 [**15**]. The GPM98B model contains some gravity data from Israel; therefore it should fit Israel better than any other global geopotential model. Early experiments using the GPM98B in Israel showed a local accuracy of approximately 10-20 cm, and differences of up to plus 2 metres in the north and minus 2 metres in the south between the Survey of Israel undulation model and the GPM98B.

CONCEPT OF AN OFFICIAL GEOID UNDULATION MODEL

Intensive utilisation of GNSS for geodetic and engineering applications necessitates the rapid development of an undulation model. Today, the efforts to develop a geoid undulation model with an accuracy level of one centimetre over the entire country demand multiple resources, much like the efforts needed to achieve a dense levelling network with an accuracy level of one centimetre. The objective of a vertical control network is to bring consistent and identical heights to all points (within the desired accuracy) obtained by every surveyor. In reality this goal cannot be achieved by means of classic levelling networks. It may be achieved by GNSS levelling, but that requires an accurate and homogenous geoid-undulation model.

In order to overcome this problem, we suggest declaring the best available geoid undulation model as the official model. The official geoid undulations model (OGUM) can be replaced from time to time by a better model. The combination of the OGUM with a vertical ellipsoidal control based on permanent GNSS stations produces a practical countrywide network of "official" orthometric heights.

Of course, the use of this concept depends on the specific accuracy needs for the orthometric height differences, the accuracy of the vertical ellipsoidal control network, and last but not least, the accuracy of the best available undulation model. There are certainly projects, for which a higher accuracy level of the orthometric control will be required. These projects do not require a nationwide accurate orthometric control system. Wherever the proposed idea is insufficient, one could use a local "Orthometric Island" as proposed by Steinberg and Papo [**13**]. However, any orthometric island system should always be regarded as a secondary control, which draws its datum from the national ellipsoidal control.

In order to test our suggestion and to estimate the accuracy of the obtained orthometric height differences, we conducted several tests in Israel. The world-wide geopotential model GPM98B was tested, in addition to the local Israeli geoid undulation model. This research is described in the following chapter.

OFFICIAL UNDULATION MODEL EXPERIMENTS

The goal of the experiment was to examine the accuracy of orthometric height differences, based on GPS measurements and the OGUM in different locations throughout Israel. Two alternative options were tested for the OGUM, a local Israeli model produced by SOI, and the aforementioned global GPM98B model. In each location, four adjacent orthometric control points (benchmarks), with known orthometric heights, were measured simultaneously using GPS. It is important to note that the chosen measured benchmarks were not used as part of the Israeli undulation model. The duration of the measurement session was 40 minutes. The six ellipsoidal height differences were processed from the GPS measurements.

By using the OGUM, we can calculate the undulation differences between the points. Equation (2) was used to compute the orthometric height differences. The existing orthometric height differences between the benchmarks are known. Therefore, a comparison between the known and computed orthometric height differences allows us to evaluate the accuracy of orthometric height differences based on the GPS measurements and the OGUM. The estimated relative accuracy of the known orthometric height differences between the benchmarks, as well as the measured ellipsoidal height differences, was approximately 1-2 cm.

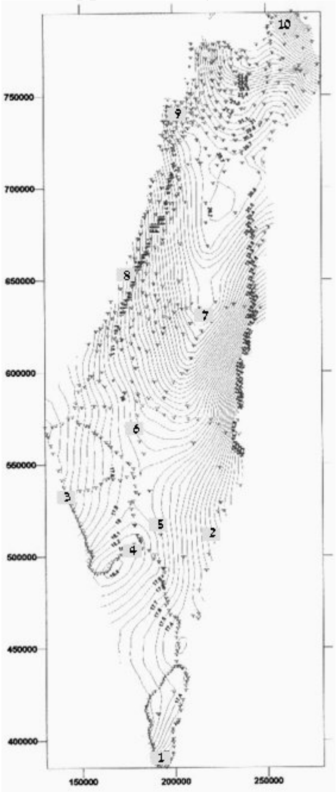


Fig. 1. Ten locations (squares) where the OGUM experiment was held in Israel, on a background of the SOI undulation map (contour line every 25cm) based on approximately 750 points (dots). The numbers denote the location names, as appearing in Table 1.

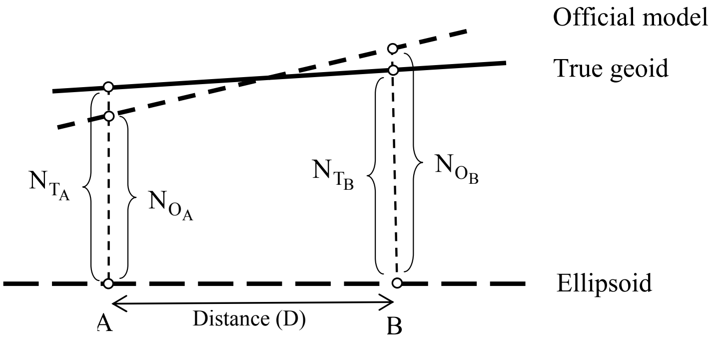
In addition, the GPS data from each point was processed relative to VRS (Virtual Reference Station) data that was created in a virtual point in the centre of the four measured benchmarks. The ellipsoidal height differences between points were computed based on the processed ellipsoidal height of each point. These ellipsoidal height differences and the calculated undulation differences between the points were also used to obtain computed orthometric height differences. The experiment was carried out in ten locations in Israel, from Eilat in the south to Kiryat-Shmona in the north (see Fig. 1). Since there is no orthometric control network in location number 5 (Machmal valley), levelling was carried out between the points to obtain the orthometric height differences. In location number 9 (The Technion, Haifa) ten points were measured (instead of four) and levelled using precise levelling. It is important to note the short distances between the points in this area.

Table 1. *The rms and the mean ppm value of the differences between the known orthometric height differences and the calculated orthometric height differences for the two OGUM alternatives. In section (a) the calculated orthometric height differences are obtained by direct GPS measurements between the benchmarks; in section (b) the calculated orthometric height differences are obtained indirectly through VRS. The maximum difference (absolute value) is presented in brackets.*

(a)				**SOI Undulations Model**		**GPM98B**	
Place	Min. dist. [km]	Max. dist. [km]	Max. ht. diff. [m]	RMS (max. difference) [cm]	Mean (max. difference) [ppm]	RMS (max. difference) [cm]	Mean (max. difference) [ppm]
1. Eilat	1.4	4.5	94.1	5.0 (6.8)	14 (23)	22.1 (35.8)	61 (105)
2. Hazeva	2.9	5.6	40.5	5.0 (8.1)	10 (20)	8.2 (13.5)	15 (26)
3. Nitzana	1.8	4.5	28.5	1.0 (1.5)	3 (5)	3.3 (5.3)	12 (17)
4.Mizpe-Ramon	0.6	3.3	49.3	1.0 (1.6)	8 (24)	4.3 (7.2)	25 (46)
5.Machmal valley	0.7	1.6	27.5	0.4 (0.9)	2 (6)	1.8 (3.5)	13 (23)
6. Beer-Sheva	1.3	4.8	15.7	2.4 (4.5)	8 (17)	2.2 (4.2)	8 (18)
7. Jerusalem	1.0	4.1	101.3	1.9 (3.0)	8 (16)	1.4 (2.6)	5 (10)
8. Tel-Aviv	1.8	7.6	15.6	1.9 (3.1)	4 (7)	1.7 (2.8)	3 (6)
9.Technion, Haifa	0.07	0.8	57.9	2.1 (4.2)	77 (427)	1.9 (4.5)	74 (458)
10.Kiryat-Shmona	0.3	1.8	215.6	1.9 (3.4)	21 (57)	2.7 (4.5)	20 (48)

(b)				**SOI Undulations Model**		**GPM98B**	
Place	Min. dist. [km]	Max. dist. [km]	Max. ht. diff. [m]	RMS (max. difference) [cm]	Mean (max. difference) [ppm]	RMS (max. difference) [cm]	Mean (max. difference) [ppm]
1. Eilat	1.4	4.5	94.1	3.9 (6.0)	10 (19)	22.2 (36.6)	62 (101)
2. Hazeva	2.9	5.6	40.5	7.4 (10.6)	15 (32)	6.7 (11.5)	14 (22)
3. Nitzana	1.8	4.5	28.5	2.8 (4.5)	7 (18)	2.3 (3.7)	7 (16)
4.Mizpe-Ramon	0.6	3.3	49.3	1.3 (2.1)	10 (26)	4.0 (6.7)	24 (48)
5.Machmal valley	0.7	1.6	27.5	1.5 (2.5)	12 (24)	3.1 (4.6)	18 (45)
6. Beer-Sheva	1.3	4.8	15.7	3.2 (4.3)	14 (24)	3.0 (4.0)	14 (25)
7. Jerusalem	1.0	4.1	101.3	2.3 (3.8)	12 (38)	2.1 (3.4)	11 (25)
8. Tel-Aviv	1.8	7.6	15.6	1.8 (2.5)	5 (7)	1.6 (2.2)	3 (5)
9.Technion, Haifa	0.07	0.8	57.9	n/a	n/a	n/a	n/a
10.Kiryat-Shmona	0.3	1.8	215.6	4.7 (7.4)	38 (62)	5.8 (9.7)	52 (99)

Table 1 presents the results, including the root mean square (rms) and the mean ppm value of differences between the known orthometric height differences, as well as the orthometric height differences based on the GPS measurements and the two alternatives for the OGUM in each area. Table 1 also shows that the Survey of Israel undulation model and the GPM98B provide orthometric height differences based on GPS measurements with a relative accuracy that is better than 25 ppm. Thus, the slope of the official geoid model with respect to the ellipsoid surface is similar to the slope of the true geoid with accuracy better than 25 mm between benchmarks located 1 km apart (see Fig 2). Slightly worse results were obtained when using the ellipsoidal height differences, which were measured indirectly by VRS post-processing. Nevertheless, VRS provided a relative accuracy that was similar to direct GPS measurements. At location number 9 (The Technion, Haifa) the mean relative accuracy was low (77 ppm for the SOI model and 74 ppm for GPM98B) due to the very short distances between the points and due to the low quality of the GPS measurements, which was caused by a poor choice of some of the points. This poor choice of antenna locations resulted in many obstacles to the satellites and a high level of multipath. In the southern locations (1, 2, 3, 4 and 5) the SOI undulation model provided better results than the GPM98B. It also obtained better results in location number 10 (Kiryat-Shmona) in the northern part of Israel. In location number 1 (Eilat) the GPM98B did not provide a sufficient accuracy level. It seems that the GPM98B suffers from the lack of data in the southern part of Israel, as a result from the special location of Eilat on the northern edge of the Red Sea.



$$[(N_{O_A} - N_{O_B}) - (N_{T_A} - N_{T_A})]/D < 25\text{ppm}$$

Fig.2. The difference between the official geoid undulation model (N_0) and the true geoid (N_T)

As mentioned above, the estimated relative accuracy of the known orthometric height differences between the benchmarks is about 1-2 cm. In two locations (numbers 5 and 9) precise levelling was carried out as part of the experiments. In those cases, the accuracy of the orthometric height differences between the points is at the 1 millimetre level. Based on these two locations, we can not conclude that the computed orthometric height differences are more accurate than in other places. However, we can assume that the known orthometric heights do not constitute a serious source of error in other places. Since the accuracy of the OGUM is fixed, the accuracy of the GPS measurements plays a major role in converting ellipsoidal height to orthometric height.

SUMMARY AND CONCLUSION

The two tested geoid models provide orthometric height differences based on GNSS measurements with relative accuracy that is better than 25 ppm. This relative accuracy is an improvement compared to the actual relative accuracy required from vertical control systems. Therefore, it is sufficient for most surveying and engineering work. The results of the experiments demonstrate the great potential of the official geoid undulation model concept. It can be used mainly in undeveloped areas and countries without a fully formed levelling network.

The most important advantage of an OGUM is its consistency. When used relatively, it can be regarded as errorless. The nominal accuracy of the orthometric height differences is mostly dependent on the accuracy of the GPS measurements. Thus, the best available model should be chosen as the OGUM. A preference should be given to a local model that best fits the existing orthometric control; otherwise a global geoid model like GPM98B may be used. But in this case it should be fitted best to as many benchmarks as possible. We found differences of up to plus 2 m (in the north) and minus 2 m (in the south), between the Israeli model and the GPM98B, over a distance of about 400 km. As the general geodetic rule mandates, it is important to know the accuracy of the chosen model, and to use it accordingly.

When considering a vertical ellipsoidal control system as part of a 3D geodetic control based on permanent GNSS stations, and while understanding that some kind of national orthometric control is still necessary, we believe that the suggested OGUM concept is a valid substitute for a national levelling network [**12**]. The height information obtained when using GNSS measurements and an OGUM creates a nationwide orthometric island. To establish a local orthometric control network, a specific point with its orthometric height, obtained using this technique, can obviously determine a datum for a local orthometric island where higher accuracy is needed.

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